2015/16-ME
MATH CP
PAPER 1

HOK YAU CLUB

HONG KONG MOCK EXAMINATION 2015/16

MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

9.00 am – 11.15 am (2¼ hours)

This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1 and 3.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

Please stick the barcode label here.

Candidate Number									

	Marker's Use Only	**Examiner's Use Only**
	Marker No.	Examiner No.
Question No.	Marks	Marks
1-2		
3-4		
5-6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
Total		

SECTION A(1) (35 marks)

1. Simplify $\frac{x^{-9}}{(x^2y^{-3})^4}$ and express your answer with positive indices. (3 marks)

2. Make n the subject of the formula $m = \frac{n-1}{n+1}$. (3 marks)

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Please stick the barcode label here.

3. A bag contains n cards, in which 12 of them are number cards and the remaining cards are letter cards. If one card is randomly drawn from the bag, the probability of getting a letter card is $\frac{3}{5}$. Find the value of n. (3 marks)

4. Sally is the manager of a logistics company. She is going to deliver the following products:

	Weight per box (kg)	Quantity (box)
Toy plane	13.8	35
Toy ship	15.5	50
Toy car	9.2	60

(a) By rounding up the weight of each box of the products to the nearest kg, estimate the total weight of the products that Sally needs to deliver.

(b) If Sally can only arrange one truck with a maximum load of 1900 kg, using the result of (a), explain why the truck can carry all the goods at one time.

(4 marks)

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5. Factorize

(a) $4x^2 - 4xy - 8y^2$,

(b) $4x^2 - 4xy - 8y^2 - 3x + 6y$.

(4 marks)

6. (a) Solve the inequality $\frac{9-4x}{5} \leq 2(x+3)$.

(b) How many integers satisfy both the inequalities $\frac{9-4x}{5} \leq 2(x+3)$ and $2x - \frac{13}{2} < 0$?

(4 marks)

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7. The marked price of a book is \$195. If the book is sold at a discount of 20%, the percentage profit is 30%. Find the cost of the book. (4 marks)

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8. In Figure 1 , $ABCD$ is a square and ABE is an equilateral triangle. CE is produced to meet AD at F.

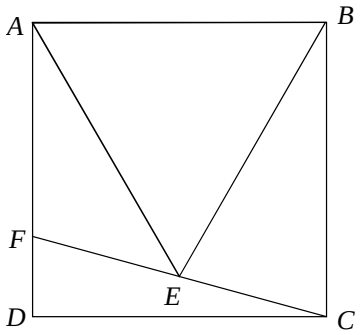

Figure 1

(a) Find $\angle AEF$.

(b) Is ΔAEF an isosceles triangle? Explain your answer.

(5 marks)

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9. The coordinates of the points P and Q are $(-3, 5)$ and $(1, k)$ respectively, where $k > 0$. P' is the reflection of P with respect to the y-axis. Q is rotated clockwise about the origin O through $90°$ to Q'.

(a) Write down the coordinates of P' and express the coordinates of Q' in terms of k.

(b) Suppose PQ is perpendicular to $P'Q'$, find the value of k.

(5 marks)

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SECTION A(2) (35 marks)

10. Ricky's expenditure $\$E$ in a month is the sum of two parts, one part is a constant and the other part varies as his income $\$I$ in that month. When $I = 16500$, $E = 16000$; when $I = 18000$, $E = 17000$.

(a) Ricky's income is $\$21000$ in a month, find his expenditure in that month. (4 marks)

(b) After deducting Ricky's monthly expenditure in a month, the remaining amount is $\frac{1}{4}$ of his monthly income, find his income in that month. (2 marks)

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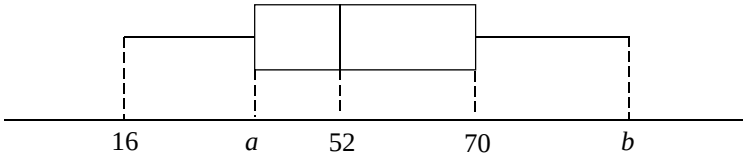
11. There are 35 students in a S5 class. The box-and-whisker diagram below shows the distribution of the scores (in marks) of the students of that class in a Mathematics Examination. It is given that the range of this distribution is 77 marks, the inter-quartile range is 32 marks and the mean score is 54 marks. Moreover, no students get the same score.



(a) Find a and b. (2 marks)

(b) It is given that all the students of that class are promoted to S6 , but three of them with Mathematics scores 25 marks, 40 marks and 69 marks are transferred to another class. On the other hand, two new students with Mathematics scores 28 marks and 52 marks are enrolled into this class.

(i) Find the mean score of the Mathematics Examination of that S6 class.

(ii) After the enrollment adjustment, one student from that class is randomly selected by the teacher to be the Mathematics subject leader. The monitor of that class claims that the probability of the score of the selected student being lower than or equal to the median is greater than the probability of the score of the selected student being higher than or equal to the median. Do you agree? Explain your answer.

(4 marks)

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12. Let $f(x) = 2x^3 + kx^2 + 5x + 6$, where k is a constant. It is given that when $f(x)$ is divided by $ax^2 - 4x - 1$, the quotient is $x - 2$ and the remainder is $bx + c$, where a, b and c are constants.

(a) Find a, b and c. (4 marks)

(b) It is given that $g(x) = f(x) - x^2 + 2x$. Factorize $g(x)$. (3 marks)

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13. $H(2, 0)$ is a point in the rectangular coordinate plane. P is a moving point in the same rectangular coordinate plane such that P keeps a distance of 5 units from H. Denote the locus of P by Γ.

(a) Write down the equation of Γ. (1 mark)

(b) Γ cuts the x-axis at two points $A(a, 0)$ and $B(b, 0)$, where $a < b$, and cuts the positive y-axis at C. Straight line L_1 passes through points A and C. Straight line L_2 passes through points B and C.

(i) Someone claims that L_1 is perpendicular to L_2. Do you agree? Explain your answer.

(ii) Straight line L_3 passes through the origin O and intersects L_2 at D such that $OD \perp BC$. Find the ratio of the area of ΔOBD to the area of quadrilateral $OACD$.

(6 marks)

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14. Figure 2 shows a solid made by cutting off a right circular cone and a hemisphere from a cylinder of height 60 cm. The base radius and the height of the removed circular cone are r cm and 24 cm respectively, and the radius of the hemisphere is r cm. It is given that the sum of the volumes of two removed parts is $\frac{1}{3}$ of the volume of the cylinder.

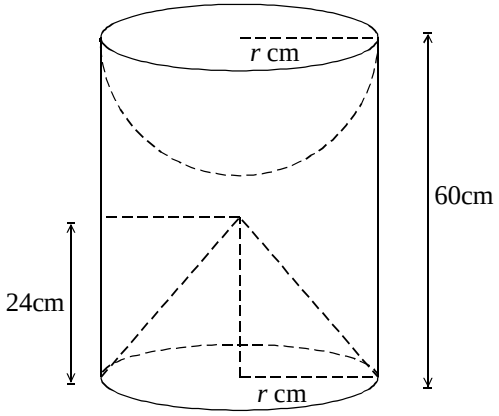

Figure 2

(a) Find r. (3 marks)

(b) Find the total surface area of the solid in terms of π. (3 marks)

(c) Suppose the solid is cut into two identical parts. Someone claims the increase in the total surface area is less than $0.3\,\text{m}^2$. Do you agree? Explain your answer. (3 marks)

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SECTION B (35 marks)

15. The table below shows the distribution of the weights of some students. It is given that the mean of the weights of students is 58 kg .

Weight (kg)	Number of students
45 – 49	2
50 – 54	6
55 – 59	x
60 – 64	5
65 – 69	5

(a) Find x and the standard deviation of the weights of students. (2 marks)

(b) It is given that the standard score of the weight of Kelly is -1.3 . Is Kelly one of the two lightest students? Explain your answer. (2 marks)

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16. 2 girls and 6 boys are randomly arranged in two rows of 4 for a group photo. Find the probabilities of the following events.

(a) Two girls are arranged next to each other. (2 marks)

(b) Two girls are arranged in the same row. (2 marks)

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17. Let p, q and r are positive real numbers such that $\log p$, $\log q$ and $\log r$ form an arithmetic sequence.

(a) Does the equation $px^2 + qx + r = 0$ have real roots? Explain your answer. (3 marks)

(b) Suppose α and $\frac{1}{\alpha}$ are the two roots of the equation $px^2 + qx + r = 0$. Find α , express the answer in the form $a + bi$, where a and b are real numbers and $i^2 = -1$. (3 marks)

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18. Let $\mathrm{f}(x) = ax^2 - 6kx + 4k^2 + 2$, where $k > \frac{1}{3}$. It is given that P is the vertex of the graph of $y = \mathrm{f}(x)$ and the graph of $y = \mathrm{f}(x)$ passes through the point $R(2k, 4k^2 + 2)$.

(a) (i) Find a.

(ii) Express the coordinates of point P in terms of k.

(4 marks)

(b) In the same rectangular coordinate system, Q is the vertex of the graph of $y = \mathrm{f}(x-k)+k$. Denote the origin by O. Someone claims that the orthocentre of ΔPQR lies inside ΔPQR. Is the claim correct? Explain your answer. (4 marks)

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19. Figure 3 shows a trapezoidal advertisement board $ABCD$ standing vertically on the horizontal ground along the east-west direction. $AD = 3\text{m}$, $BC = 4\text{m}$, $DC = 6\text{m}$, $AD \perp DC$ and $BC \perp DC$. When the sun shines from N40°W with an angle of elevation $35°$, the shadow of the advertisement board on the horizontal ground is $DCFE$.

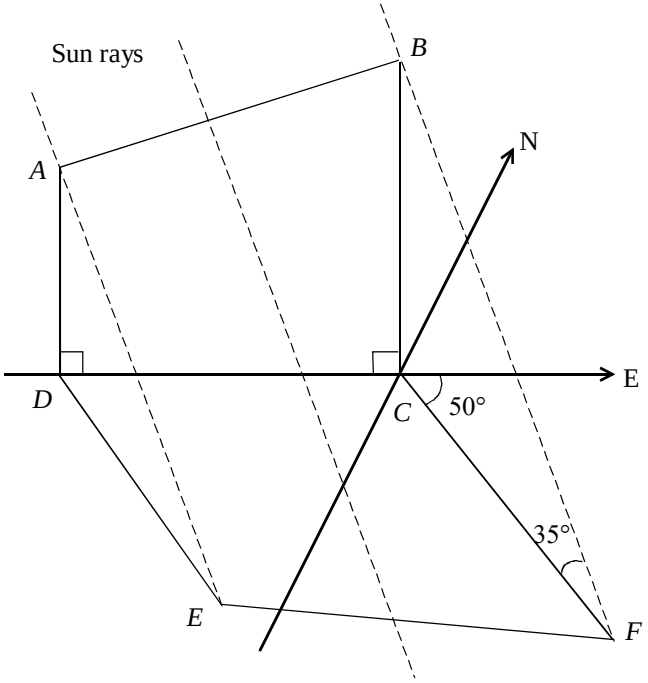

Figure 3

(a) Find the area of the shadow $DCFE$. (5 marks)

(b) Suppose the sun shines from Nθ°W, where $40 < \theta < 90$, and its angle of elevation is still $35°$. State with reasons whether the area of shadow of the advertisement board on the horizontal ground is greater than, less than or equal to the area obtained in (a). (2 marks)

(c) A man with the height 1.8 m walks from E to CF along the shortest path. When he walked x m, he finds that the sunlight is just overhead (i.e. he is in the shadow of the advertisement board). Find x. (6 marks)

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