試卷一

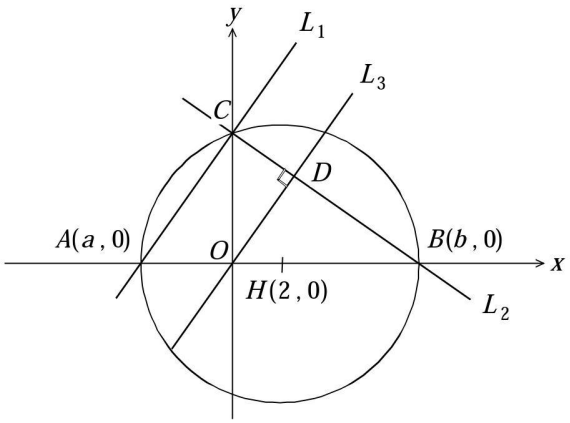
解	分	備註
1. $\dfrac{x^{-9}}{(x^2y^{-3})^4}$		
$=\dfrac{x^{-9}}{x^8y^{-12}}$	1M	給 $(ab)^m=a^mb^m$ 或 $(a^m)^n=a^{mn}$
$=\dfrac{y^{12}}{x^{8-(-9)}}$	1M	給 $\dfrac{1}{c^{-p}}=c^p$ 或 $\dfrac{c^p}{c^q}=\dfrac{1}{c^{q-p}}$
$=\dfrac{y^{12}}{x^{17}}$	1A	
	---------(3)	
2. $m=\dfrac{n-1}{n+1}$		
$m(n+1)=n-1$		
$mn+m=n-1$	1M	
$mn-n=-m-1$	1M	給將 n 放在一邊
$n(m-1)=-m-1$		
$n=\dfrac{-m-1}{m-1}$	1A	給 $n=-\dfrac{m+1}{m-1}$ 或等價
	---------(3)	
3. $\dfrac{n-12}{n}=\dfrac{3}{5}$	1M	給 $\dfrac{n-12}{n}=P$
$5n-60=3n$	1A	
$2n=60$		
$n=30$	1A	
$\dfrac{12}{n}=\dfrac{2}{5}$	1M	給 $\dfrac{12}{n}=P$
$60=2n$	1A	
$n=30$	1A	
	---------(3)	
4. (a) 總重量 $=14\times35+16\times50+10\times60$	1M	至少對兩項
$=1890$ (kg)	1A	
(b) 由於上捨入的總重量 $=1890$ kg		
<1900 kg	1M	
因此原有產品的總重量 <1900 kg		
故該貨車能夠一次過運載全部的產品。	1	必須顯示理由
	---------(4)	

	解	分	備註
5.	(a) $4x^2-4xy-8y^2$		
	$=4(x^2-xy-2y^2)$	1M	給 $4(a+b+c)$
	$=4(x-2y)(x+y)$	1A	或等價
	(b) $4x^2-4xy-8y^2-3x+6y$		
	$=4(x-2y)(x+y)-3(x-2y)$	1M	給利用 (a) 的結果
	$=(x-2y)[4(x+y)-3]$		
	$=(x-2y)(4x+4y-3)$	1A	或等價
		--------(4)	
6.	(a) $\frac{9-4x}{5}\le 2(x+3)$		
	$9-4x\le 10(x+3)$		
	$9-4x\le 10x+30$		
	$-4x-10x\le 30-9$	1M	給將 x 放在一邊
	$-14x\le 21$		
	$x\ge -\frac{3}{2}$	1A	
	(b) $2x-\frac{13}{2}<0$		
	$2x<\frac{13}{2}$		
	$x<\frac{13}{4}$	1A	
	藉 (a)，可得 $-\frac{3}{2}\le x<\frac{13}{4}$ 。		
	因此，有 5 個整數能同時滿足不等式組。	1A	
		--------(4)	
7.	設成本為 $\$x$。		
	按題意，得		
	$x(1+30\%)=195\times 80\%$	1M+1M +1A	1M 給 $x(1+30\%)$ +1M 給 $195\times 80\%$
	$x=195\times 80\%\div 130\%$		
	$=195\times\frac{80}{100}\times\frac{100}{130}$		
	$=120$	1A	
	因此，這本書的成本為 $\$120$ 。		
		--------(4)	

解	分	備註
8. (a) $\angle EBC = 90° - 60° = 30°$	1A	
$\because\ BE = BC$		
$\therefore\ \angle BEC = \dfrac{180° - \angle EBC}{2}$	1M	
$= \dfrac{180° - 30°}{2}$		
$= 75°$		
$\therefore\ \angle AEF = 180° - \angle BEC - \angle AEB$		
$= 180° - 75° - 60°$		
$= 45°$	1A	
(b) 注意 $\angle AFE > 90°$ 。		
又 $\angle FAE = 90° - 60° = 30°$		
$\therefore\ \angle FAE \neq \angle AEF$	1M	
$\therefore\ FA \neq FE$		
$\therefore\ \Delta AEF$ 不是一等腰三角形。	1A	必須顯示理由
	--------(5)	
9. (a) P' 點坐標為 $(3, 5)$	1A	
Q' 點坐標為 $(k, -1)$	1A	
(b) PQ 的斜率 $= \dfrac{k-5}{1+3}$	1M	給任何一項
$= \dfrac{k-5}{4}$		
$P'Q'$ 的斜率 $= \dfrac{5+1}{3-k}$		
$= \dfrac{6}{3-k}$		
$\because\ PQ \perp P'Q'$		
$\therefore\ \dfrac{k-5}{4} \cdot \dfrac{6}{3-k} = -1$	1M	
$6k - 30 = 4k - 12$		
$2k = 18$		
$k = 9$	1A	
	--------(5)	

解	分	備註
10. (a) $E=a+bI$ ，其中 a 及 b 均為非零的常數。	1A	
代入 $I=16500$ 及 $E=16000$		
得 $16000=a+16500b$ ------(1)	1M	給任何一項代入
代入 $I=18000$ 及 $E=17000$		
得 $17000=a+18000b$ ------(2)		
解 (1) 及 (2) 兩式，得 $a=5000$ ， $b=\frac{2}{3}$	1A	給兩項正確
$\therefore\ E=5000+\frac{2}{3}I$		
當 $I=21000$ 時，		
$E=5000+\frac{2}{3}\times 21000$		
$=19000$	1A	
因此，他該月的支出為 \$19000。		
	---------(4)	
(b) 設該每月的收入為 $\$I$ 。		
則 $5000+\frac{2}{3}I=\frac{3}{4}I$	1M	給 $a+bI=\frac{3}{4}I$
$5000=\frac{1}{12}I$		
$I=60000$	1A	
因此，他該月的收入為 \$60000。		
設該月的收入為 $\$I$ 。		
則 $I-(5000+\frac{2}{3}I)=\frac{1}{4}I$	1M	給 $I-(a+bI)=\frac{1}{4}I$
$\frac{1}{12}I=5000$		
$I=60000$	1A	
因此，他該月的收入為 \$60000。		
	---------(2)	
11. (a) $70-a=32$	1M	給任何一項
$a=38$		
$b-16=77$		兩項全對
$b=93$	1A	
	---------(2)	
(b) (i) 新的平均值		
$=\frac{54\times 35-25-40-69+28+52}{35-1}$	1M	
$=54$ (分)	1A	
(ii) 留意 25，40 及 28 小於中位數 52，又 69 大於中位數 52。		
因此，調走 25，40 及 69 這三個數據，又加入 28 這個數據後，中位數保持不變。另外加入 52 這個數據，也不會改變中位數的值。	1M	
所以，低於或等於中位數的數據之個數，與高於或等於中位數的數據之個數相等 (均為 18)。		接受兩者均等於 17
因此，不同意班長的宣稱。	1A	必須顯示理由
	---------(4)	

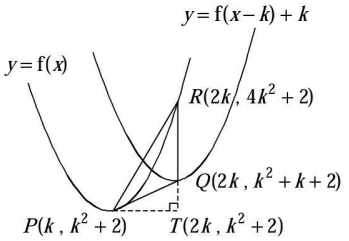
解	分	備註
12. (a) $2x^3+kx^2+5x+6\equiv(ax^2-4x-1)(x-2)+bx+c$	1M	接受用「=」連結
藉比較 x^3 項、x 項及常數項，得		
$a=2$	1A	
$7+b=5$	1M	給任何一項
$b=-2$		兩項全對
$2+c=6$		
$c=4$	1A	
藉比較 x^3 項及 x^2 項，得		
$a=2$	1A	
$k=-4-4=-8$		
$2x^2-4x-1\overline{)2x^3-8x^2+5x+6}$ $x-2$；$2x^3-4x^2-x$；$-4x^2+6x+6$；$-4x^2+8x+2$；$-2x+4$	1M	
$\therefore\ b=-2$ ， $c=4$	1A	
	---------(4)	
(b) $f(x)=(2x^2-4x-1)(x-2)-2x+4$		
$g(x)=(2x^2-4x-1)(x-2)-2x+4-x^2+2x$		
$=(2x^2-4x-1)(x-2)-(x+2)(x-2)$	1A	
$=(x-2)(2x^2-5x-3)$	1M	給公因式 $x-2$
$=(x-2)(x-3)(2x+1)$	1A	
$f(x)=(2x^2-4x-1)(x-2)-2x+4$		
$=2x^3-8x^2+5x+6$		
$g(x)=2x^3-9x^2+7x+6$	1A	
$=(x-2)(2x^2-5x-3)$	1M	或 $(x-3)(2x^2-3x-2)$
$=(x-2)(x-3)(2x+1)$	1A	
	---------(3)	

解	分	備註
13. 		
(a) Γ 的方程為 $(x-2)^2+y^2=25$	1A ---------(1)	$x^2+y^2-4x-21=0$
(b) (i) $\because$ AB 為圓的直徑。 $\therefore$ $\angle ACB=90°$ $\therefore$ $L_1 \perp L_2$	1M	
因此，同意該宣稱。	1A	必須顯示理由
(ii) $a=-3$ ， $b=7$ $OA=3$ ， $OB=7$ $\Delta OBD \sim \Delta ABC$	1A	兩項全對
$\therefore$ ΔOBD 的面積 ： ΔABC 的面積 $=7^2:10^2$ $=49:100$	1M	
$\therefore$ ΔOBD 的面積 ：四邊形 $OACD$ 的面積 $=49:51$	1M+1A ---------(6)	

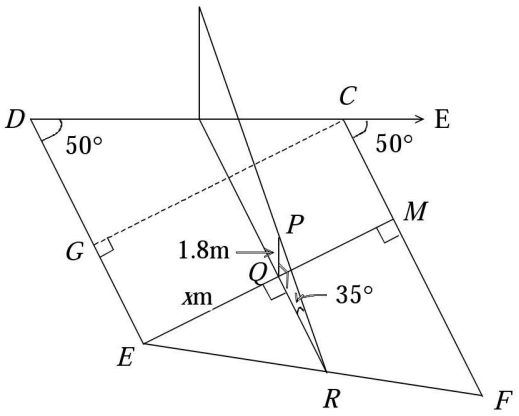
解	分	備註
14. (a) $\frac{1}{3}\pi r^2 \times 24 + \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{1}{3}(\pi r^2 \times 60)$	1M+1A	1M 給任意兩項對
$8 + \frac{2}{3}r = 20$		
$r = 18$	1A	
	---------(3)	
(b) 圓錐體的斜高		
$= \sqrt{24^2 + 18^2}$	1M	
$= 30$ (cm)		
所求立體的總表面積		
$= 2\pi \times 18 \times 60 + \pi \times 18 \times 30 + 2\pi \times 18^2$	1M	
$= 3348\pi$ (cm^2)	1A	
	---------(3)	
(c) 所增加的總表面積		
$= 2(60 \times 36 - \frac{1}{2}\pi \times 18^2 - \frac{1}{2} \times 24 \times 36)$ cm^2	1M+1A	
≈ 2438.12398 cm^2		
$= 0.243812398$ m^2		
< 0.3 m^2		
因此，同意該宣稱。	1A	必須顯示理由
	---------(3)	

解	分	備註
15. (a) $\dfrac{47\times 2+52\times 6+57x+62\times 5+67\times 5}{18+x}=58$		
$x=7$	1A	
標準差		
≈ 6.164414003		
≈ 6.16 (kg)	1A	接受答案準確至 6.16 kg
	--------(2)	
(b) 嘉琪的體重		
$\approx 58-1.3\times 6.164414003$ (kg)	1M	
≈ 49.9862618 (kg)		
> 49.5 (kg)		
因此，嘉琪不是這些學生中最輕的兩名之一。	1A	必須顯示理由
留意		
$\dfrac{49.5-58}{6.164414003}$	1M	
≈ -1.37888208		
< -1.3		
因此，嘉琪不是這些學生中最輕的兩名之一。	1A	必須顯示理由
	--------(2)	
16. (a) 所求概率		
$=\dfrac{6\times P_2^2\times P_6^6}{P_8^8}$	1M	給分子
$=\dfrac{3}{14}$	1A	接受答案準確至 0.214
所求概率		
$=\dfrac{C_2^6\times P_3^3\times P_2^2\times P_4^4\times 2}{P_8^8}$	1M	給分子
$=\dfrac{3}{14}$	1A	接受答案準確至 0.214
	--------(2)	
(b) 所求概率		
$=\dfrac{12\times P_2^2\times P_6^6}{P_8^8}$	1M	給分子
$=\dfrac{3}{7}$	1A	接受答案準確至 0.429
所求概率		
$=\dfrac{C_2^6\times P_4^4\times P_4^4\times 2}{P_8^8}$	1M	給分子
$=\dfrac{3}{7}$	1A	接受答案準確至 0.429
	--------(2)	

解	分	備註
17. (a) ∵ $\log p$ ， $\log q$ ，$\log r$ 成等差數列， ∴ $\log r-\log q=\log q-\log p$ $\log p+\log r=2\log q$ $q^2=pr$	1A	
對方程 $px^2+qx+r=0$ ， $\Delta=q^2-4pr$ $=pr-4pr$ $=-3pr$ $<0 \quad (p, r>0)$	1M	
因此，方程 $px^2+qx+r=0$ 不存在實數根。	1A ---------(3)	必須顯示理由
(b) $(\alpha)(\frac{1}{\alpha})=\frac{r}{p}$ $1=\frac{r}{p}$ $p=r$		
又由 $q^2=pr$ ， 得 $p=q=r \quad (q>0)$	1A	
代入方程 $px^2+qx+r=0$ ， 得 $x^2+x+1=0 \quad (p=q=r\neq 0)$	1M	
$x=\frac{-1\pm\sqrt{-3}}{2}$ $=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$		
因此，$\alpha=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ 。	1A ---------(3)	或 $\alpha=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

解	分	備註
18. (a) (i) $f(x)=ax^2-6kx+4k^2+2$		
$f(2k)=4ak^2-12k^2+4k^2+2=4k^2+2$	1M	
$4ak^2=12k^2$		
$a=3$ $(\because k\neq 0)$	1A	
(ii) $f(x)=3x^2-6kx+4k^2+2$		
$=3(x^2-2kx+k^2-k^2)+4k^2+2$	1M	
$=3(x-k)^2+k^2+2$		
$\therefore$ P 點坐標為 (k, k^2+2) 。	1A	
	--------(4)	
(b) Q 點坐標為 $(2k, k^2+k+2)$	1M	
留意 RQ 為一鉛垂線。		
再留意		
$4k^2+2-(k^2+k+2)$		
$=3k^2-k$		
$=k(3k-1)$		
>0 $(\because k>\frac{1}{3})$		
因此， R 點位於 Q 點之上。	1A	必須顯示理由
作 PT 垂直 RQ 或 RQ 的延長線使垂足為 T。		
則 T點坐標為 $(2k, k^2+2)$		
因此， T 點位於 Q 點之下。		
即 T 點位於線段 RQ 的延長線上。	1M	
故 ΔPQR 的垂心位於 ΔPQR 之外。		
因此，該宣稱不正確。	1A	必須顯示理由
PQ 的斜率 $=\frac{k^2+k+2-(k^2+2)}{2k-k}$		
$=1$		
>0		
故 $\angle RQP$ 為一鈍角。	1M	
所以， ΔPQR 的垂心位於 ΔPQR 之外。		
因此，該宣稱不正確。	1A	必須顯示理由
	--------(4)	

解	分	備註
19. (a) 在 rt.ΔADE 中， $DE=\dfrac{3}{\tan 35°}$ (m) ≈ 4.28444402 (m) 在 rt.ΔBCF 中， $CF=\dfrac{4}{\tan 35°}$ (m) ≈ 5.712592027 (m) 由 E 作 $EM\perp CF$ 使垂足為 M。 則 $EM=6\sin 50°$ (m) ≈ 4.596266659 (m) 投影 $DCFE$ 的面積 $=(\dfrac{3}{\tan 35°}+\dfrac{4}{\tan 35°})\times\dfrac{6\sin 50°}{2}$ ≈ 22.97452173 (m) ≈ 23.0 (m^2)	1M 1M 1M+1A 1A	給任何一項 $(4.28444402+5.712592027)$ $\times\dfrac{4.596266659}{2}$ $=22.97452174$ 接受答案準確至 23.0 m^2
由 E 作 $EY//DC$ 使之交 CF 於 Y。 則 $EY=DC=6$ 投影 $DCFE$ 的面積 = 平行四邊形 $DCYE$ 的面積 +ΔEYF 的面積 $=6\times\dfrac{3}{\tan 35°}\times\sin 50°$ $+\dfrac{1}{2}(\dfrac{4}{\tan 35°}-\dfrac{3}{\tan 35°})\times 6\sin(180°-50°)$ ≈ 22.97452173 ≈ 23.0 (m^2)	 1M +1M +1A 1A	 1M 給兩項面積的和 +1M 給任一項對 接受答案準確至 23.0 m^2
連 CE。在 ΔDCE 中，藉餘弦公式，可得 $CE^2=DC^2+DE^2-2(DC)(DE)\cos\angle CDE$ $CE^2=6^2+(\dfrac{3}{\tan 35°})^2-2(6)(\dfrac{3}{\tan 35°})\cos 50°$ ≈ 21.3086102 $CE\approx 4.616125019$ (m) 藉正弦公式，可得 $\dfrac{\sin\angle DCE}{DE}=\dfrac{\sin\angle CDE}{CE}$ $\dfrac{\sin\angle DCE}{4.28444402}\approx\dfrac{\sin 50°}{4.616125019}$ $\angle DCE\approx 45.31650667°$ $\angle ECF\approx 180°-50°-45.31650667°$ $=84.68349333°$ 投影 $DCFE$ 的面積 =ΔDCE 的面積 +ΔCEF 的面積 $\approx\dfrac{1}{2}\times 6\times 4.28444402\sin 50°$ $+\dfrac{1}{2}\times 4.616125019\times 5.712592027\sin 84.68349333°$ ≈ 22.97452173 ≈ 23.0 (m^2)	1M 1M +1A 1A	給任何一項 接受答案準確至 23.0 m^2
	--------(5)	

解	分	備註
(b) 投影面積 $= (DE + CF) \times \dfrac{DC \sin(90° - \theta°)}{2}$		
若 $40 < \theta < 90$ ，則 $90° - \theta°$ 小於 $50°$ 。	1M	
由於 DE ， CF 及 DC 保持不變， 而 $\sin(90° - \theta°) < \sin 50°$ ， 因此，投影面積小於 (a) 所求得的面積。	1A ---------(2)	必須顯示理由
(c)  		
最短路徑為 EM 。	1M	
在 rt.ΔPQR 中， $QR = \dfrac{1.8}{\tan 35°}$	1M	$QR \approx 2.570666412$
$MF = CF - CM$ $= CF - GE$ $= CF - (DE - DG)$ $= \dfrac{4}{\tan 35°} - (\dfrac{3}{\tan 35°} - 6\cos 50°)$ ≈ 5.284873665	1M	
$\Delta EQR \sim \Delta EMF$ $\dfrac{EQ}{EM} = \dfrac{QR}{MF}$	1M	
$\dfrac{x}{6\sin 50°} \approx \dfrac{\frac{1.8}{\tan 35°}}{5.284873665}$	1A	$\dfrac{x}{6\sin 50°} \approx \dfrac{2.570666412}{5.284873665}$
$x \approx 2.235714431$ ≈ 2.24	1A ---------(6)	接受答案準確至 2.24

Paper 1

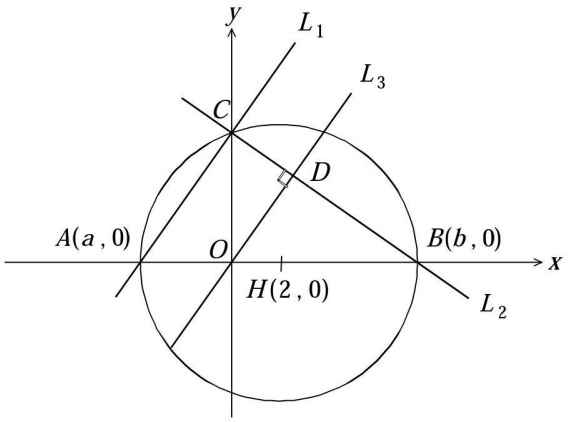
	Solution	Marks	Remarks
1.	$\dfrac{x^{-9}}{(x^2y^{-3})^4}$		
	$=\dfrac{x^{-9}}{x^8y^{-12}}$	1M	for $(ab)^m=a^mb^m$ or $(a^m)^n=a^{mn}$
	$=\dfrac{y^{12}}{x^{8-(-9)}}$	1M	for $\dfrac{1}{c^{-p}}=c^p$ or $\dfrac{c^p}{c^q}=\dfrac{1}{c^{q-p}}$
	$=\dfrac{y^{12}}{x^{17}}$	1A	
		---------(3)	
2.	$m=\dfrac{n-1}{n+1}$		
	$m(n+1)=n-1$		
	$mn+m=n-1$	1M	
	$mn-n=-m-1$	1M	for putting n on one side
	$n(m-1)=-m-1$		
	$n=\dfrac{-m-1}{m-1}$	1A	for $n=-\dfrac{m+1}{m-1}$ or equivalent
		---------(3)	
3.	$\dfrac{n-12}{n}=\dfrac{3}{5}$	1M	for $\dfrac{n-12}{n}=P$
	$5n-60=3n$	1A	
	$2n=60$		
	$n=30$	1A	
	$\dfrac{12}{n}=\dfrac{2}{5}$	1M	for $\dfrac{12}{n}=P$
	$60=2n$	1A	
	$n=30$	1A	
		---------(3)	
4.	(a) The estimated total weight $=14\times35+16\times50+10\times60$	1M	at least two terms correct
	$=1890$ (kg)	1A	
	(b) The estimated total weight $=1890$ kg		
	<1900 kg	1M	
	Note that the actual total weight is less than 1900 kg .		
	Thus, the truck can carry all the goods at one time .	1	f.t.
		---------(4)	

Solution	Marks	Remarks
5. (a) $4x^2-4xy-8y^2$		
$=4(x^2-xy-2y^2)$	1M	for $4(a+b+c)$
$=4(x-2y)(x+y)$	1A	or equivalent
(b) $4x^2-4xy-8y^2-3x+6y$		
$=4(x-2y)(x+y)-3(x-2y)$	1M	for using the result of (a)
$=(x-2y)[4(x+y)-3]$		
$=(x-2y)(4x+4y-3)$	1A	or equivalent
	---------(4)	
6. (a) $\frac{9-4x}{5}\le 2(x+3)$		
$9-4x\le 10(x+3)$		
$9-4x\le 10x+30$		
$-4x-10x\le 30-9$	1M	for putting x on one side
$-14x\le 21$		
$x\ge -\frac{3}{2}$	1A	
(b) $2x-\frac{13}{2}<0$		
$2x<\frac{13}{2}$		
$x<\frac{13}{4}$	1A	
By (a), we have $-\frac{3}{2}\le x<\frac{13}{4}$.		
Thus, there are 5 integers satisfy both the inequalities .	1A	
	---------(4)	
7. Let $\$x$ be the cost of the book.		
$x(1+30\%)=195\times 80\%$	1M+1M+1A	1M for $x(1+30\%)$ +1M for $195\times 80\%$
$x=195\times 80\%\div 130\%$		
$=195\times\frac{80}{100}\times\frac{100}{130}$		
$=120$	1A	
Thus, the cost of the book is $\$120$.		
	---------(4)	

Solution	Marks	Remarks
8. (a) $\angle EBC = 90° - 60° = 30°$	1A	
$\because\ BE = BC$		
$\therefore\ \angle BEC = \dfrac{180° - \angle EBC}{2}$	1M	
$= \dfrac{180° - 30°}{2}$		
$= 75°$		
$\therefore\ \angle AEF = 180° - \angle BEC - \angle AEB$		
$= 180° - 75° - 60°$		
$= 45°$	1A	
(b) Note that $\angle AFE > 90°$.		
Also, $\angle FAE = 90° - 60° = 30°$		
$\therefore\ \angle FAE \neq \angle AEF$	1M	
$\therefore\ FA \neq FE$		
$\therefore\ \Delta AEF$ is not an isosceles triangle .	1A	f.t.
	---------(5)	
9. (a) The coordinates of P' are $(3\ ,\ 5)$	1A	
The coordinates of Q' are $(k\ ,\ -1)$	1A	
(b) The slope of $PQ = \dfrac{k-5}{1+3}$	1M	either one
$= \dfrac{k-5}{4}$		
The slope of $P'Q' = \dfrac{5+1}{3-k}$		
$= \dfrac{6}{3-k}$		
$\because\ PQ \perp P'Q'$		
$\therefore\ \dfrac{k-5}{4} \cdot \dfrac{6}{3-k} = -1$	1M	
$6k - 30 = 4k - 12$		
$2k = 18$		
$k = 9$	1A	
	---------(5)	

		Solution	Marks	Remarks
10.	(a)	Let $E = a + bI$, where a and b are non-zero constants. Sub. $I = 16500$ and $E = 16000$ we have $16000 = a + 16500b$ ------(1) Sub. $I = 18000$ and $E = 17000$ we have $17000 = a + 18000b$ ------(2) Solving (1) and (2), we get $a = 5000$ and $b = \frac{2}{3}$ $\therefore\ E = 5000 + \frac{2}{3}I$ When $I = 21000$, $E = 5000 + \frac{2}{3} \times 21000 = 19000$ Thus, the required expenditure in that month is \$19000 .	1A 1M 1A 1A ---------(4)	for either substitution for both correct
	(b)	Let \$$I$ be his income in that month . then $5000 + \frac{2}{3}I = \frac{3}{4}I$ $5000 = \frac{1}{12}I$ $I = 60000$ Thus, the required income in that month is \$60000 .	1M 1A	for $a + bI = \frac{3}{4}I$
		Let \$$I$ be his income in that month . then $I - (5000 + \frac{2}{3}I) = \frac{1}{4}I$ $\frac{1}{12}I = 5000$ $I = 60000$ Thus, the required income in that month is \$60000 .	1M 1A ---------(2)	for $I - (a + bI) = \frac{1}{4}I$
11.	(a)	$70 - a = 32$ $a = 38$ $b - 16 = 77$ $b = 93$	1M 1A ---------(2)	either one for both correct
	(b)	(i) The mean $= \frac{54 \times 35 - 25 - 40 - 69 + 28 + 52}{35 - 1}$ $= 54$ (marks)	1M 1A	
		(ii) Note that 25, 40 and 28 are less than the median 52, while 69 is greater than the median 52. Therefore, when data 25, 40 and 69 are removed and datum 28 is added, the median remains unchanged . When datum 52 is added, the median also remains unchanged. Therefore, the number of data that lower than or equal to the median is the same as the number of data that higher than or equal to the median (both are 18). Thus, the claim is disagreed .	1M 1A ---------(4)	accept both are 17 f.t.

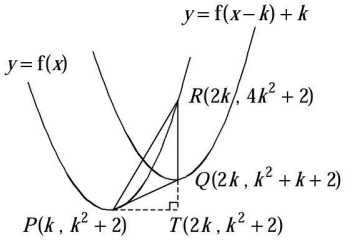
Solution	Marks	Remarks
12. (a) $2x^3+kx^2+5x+6\equiv(ax^2-4x-1)(x-2)+bx+c$	1M	accept using ' = '
By comparing the coefficients of x^3, x and the constant terms, we have		
$a=2$	1A	
$7+b=5$	1M	either one for both correct
$b=-2$		
$2+c=6$		
$c=4$	1A	
By comparing the coefficients of x^3 and x, we have		
$a=2$	1A	
$k=-4-4=-8$		
$x-2$	1M	
$2x^2-4x-1\overline{)2x^3-8x^2+5x+6}$		
$2x^3-4x^2-x$		
$-4x^2+6x+6$		
$-4x^2+8x+2$		
$-2x+4$		
$\therefore\ b=-2$ and $c=4$	1A	
	---------(4)	
(b) $f(x)=(2x^2-4x-1)(x-2)-2x+4$		
$g(x)=(2x^2-4x-1)(x-2)-2x+4-x^2+2x$		
$=(2x^2-4x-1)(x-2)-(x+2)(x-2)$	1A	
$=(x-2)(2x^2-5x-3)$	1M	for factor $x-2$
$=(x-2)(x-3)(2x+1)$	1A	
$f(x)=(2x^2-4x-1)(x-2)-2x+4$		
$=2x^3-8x^2+5x+6$		
$g(x)=2x^3-9x^2+7x+6$	1A	
$=(x-2)(2x^2-5x-3)$	1M	or $(x-3)(2x^2-3x-2)$
$=(x-2)(x-3)(2x+1)$	1A	
	---------(3)	

Solution	Marks	Remarks
13. 		
(a) The equation of Γ is $(x-2)^2+y^2=25$.	1A ---------(1)	$x^2+y^2-4x-21=0$
(b) (i) $\because$ AB is a diameter of the circle . $\therefore$ $\angle ACB=90°$ $\therefore$ $L_1 \perp L_2$	1M	
Thus, the claim is agreed .	1A	f.t.
(ii) $a=-3$, $b=7$ $OA=3$, $OB=7$ $\Delta OBD \sim \Delta ABC$	1A	for both correct
$\therefore$ Area of ΔOBD : area of ΔABC $=7^2:10^2$ $=49:100$	1M	
$\therefore$ Area of ΔOBD : area of quadrilateral $OACD$ $=49:51$	1M+1A ---------(6)	

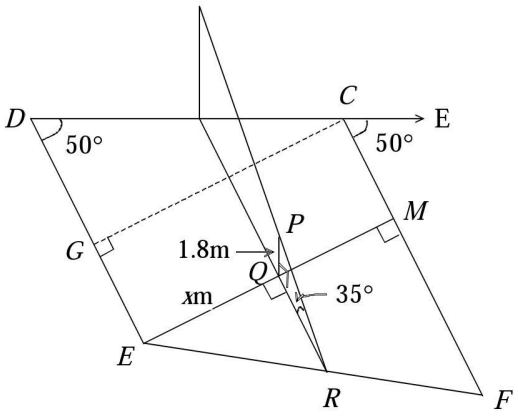
	Solution	Marks	Remarks
14. (a)	$\frac{1}{3}\pi r^2 \times 24 + \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{1}{3}(\pi r^2 \times 60)$	1M+1A	1M for any two terms correct
	$8 + \frac{2}{3}r = 20$		
	$r = 18$	1A	
		---------(3)	
(b)	The slant height of the right circular cone		
	$= \sqrt{24^2 + 18^2}$	1M	
	$= 30$ (cm)		
	The total surface area of the solid		
	$= 2\pi \times 18 \times 60 + \pi \times 18 \times 30 + 2\pi \times 18^2$	1M	
	$= 3348\pi$ (cm^2)	1A	
		---------(3)	
(c)	The increase in the total surface area		
	$= 2(60 \times 36 - \frac{1}{2}\pi \times 18^2 - \frac{1}{2} \times 24 \times 36)$ cm^2	1M+1A	
	≈ 2438.12398 cm^2		
	$= 0.243812398$ m^2		
	< 0.3 m^2		
	Thus, the claim is agreed .	1A	f.t.
		---------(3)	

		Solution	Marks	Remarks
15.	(a)	$\dfrac{47\times 2+52\times 6+57x+62\times 5+67\times 5}{18+x}=58$		
		$x=7$	1A	
		The standard deviation		
		≈ 6.164414003		
		≈ 6.16 (kg)	1A	r.t. 6.16 kg
			---------(2)	
	(b)	Kelly's weight		
		$\approx 58-1.3\times 6.164414003$ (kg)	1M	
		≈ 49.9862618 (kg)		
		> 49.5 (kg)		
		Thus, Kelly is not one of the two lightest students .	1A	f.t.
		Note that		
		$\dfrac{49.5-58}{6.164414003}$	1M	
		≈ -1.37888208		
		< -1.3		
		Thus, Kelly is not one of the two lightest students .	1A	f.t.
			---------(2)	
16.	(a)	The required probability		
		$=\dfrac{6\times P_2^2\times P_6^6}{P_8^8}$	1M	for numerator
		$=\dfrac{3}{14}$	1A	r.t. 0.214
		The required probability		
		$=\dfrac{C_2^6\times P_3^3\times P_2^2\times P_4^4\times 2}{P_8^8}$	1M	for numerator
		$=\dfrac{3}{14}$	1A	r.t. 0.214
			---------(2)	
	(b)	The required probability		
		$=\dfrac{12\times P_2^2\times P_6^6}{P_8^8}$	1M	for numerator
		$=\dfrac{3}{7}$	1A	r.t. 0.429
		The required probability		
		$=\dfrac{C_2^6\times P_4^4\times P_4^4\times 2}{P_8^8}$	1M	for numerator
		$=\dfrac{3}{7}$	1A	r.t. 0.429
			---------(2)	

Solution	Marks	Remarks
17. (a) $\because$ $\log p$, $\log q$, $\log r$ are in arithmetic sequence $\therefore$ $\log r - \log q = \log q - \log p$ $\log p + \log r = 2\log q$ $q^2 = pr$	1A	
For equation $px^2 + qx + r = 0$, $\Delta = q^2 - 4pr$ $= pr - 4pr$ $= -3pr$ $< 0 \quad (p, r > 0)$	1M	
Thus, the equation $px^2 + qx + r = 0$ has no real roots.	1A ---------(3)	f.t.
(b) $(\alpha)(\frac{1}{\alpha}) = \frac{r}{p}$ $1 = \frac{r}{p}$ $p = r$		
Also, $q^2 = pr$ so $p = q = r \quad (q > 0)$	1A	
Sub. into equation $px^2 + qx + r = 0$, we have $x^2 + x + 1 = 0 \quad (p = q = r \neq 0)$	1M	
$x = \frac{-1 \pm \sqrt{-3}}{2}$ $= -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$		
Thus, $\alpha = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$.	1A ---------(3)	or $\alpha = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

Solution	Marks	Remarks
18. (a) (i) $f(x) = ax^2 - 6kx + 4k^2 + 2$		
$f(2k) = 4ak^2 - 12k^2 + 4k^2 + 2 = 4k^2 + 2$	1M	
$4ak^2 = 12k^2$		
$a = 3 \qquad (\because\ k \neq 0)$	1A	
(ii) $f(x) = 3x^2 - 6kx + 4k^2 + 2$		
$= 3(x^2 - 2kx + k^2 - k^2) + 4k^2 + 2$	1M	
$= 3(x-k)^2 + k^2 + 2$		
$\therefore$ The coordinates of P are (k, k^2+2) .	1A	
	---------(4)	
(b) The coordinates of Q are $(2k, k^2+k+2)$	1M	
Note that RQ is a vertical line.		
Moreover,		
$4k^2 + 2 - (k^2 + k + 2)$		
$= 3k^2 - k$		
$= k(3k-1)$		
$> 0 \qquad (\because\ k > \frac{1}{3})$		
Thus, point R lies above point Q .	1A	f.t.
Construct PT perpendicular to RQ or RQ produced such that T is the foot of perpendicular .		
then the coordinates of point T are $(2k, k^2+2)$		
Thus, point T lies below point Q .		
i.e. point T lies on the produced of line segment RQ.	1M	
Therefore, the orthocenter of ΔPQR lies outside ΔPQR .		
Thus, the claim is disagreed .	1A	f.t.
The slope of $PQ = \frac{k^2+k+2-(k^2+2)}{2k-k}$		
$= 1$		
> 0		
i.e. $\angle RQP$ is an obtuse angle .	1M	
Therefore, the orthocenter of ΔPQR lies outside ΔPQR .		
Thus, the claim is disagreed .	1A	f.t.
	---------(4)	

Solution	Marks	Remarks
19. (a) In rt.ΔADE,		
$DE = \dfrac{3}{\tan 35°}$ (m)	1M	
≈ 4.28444402 (m)		either one
In rt.ΔBCF,		
$CF = \dfrac{4}{\tan 35°}$ (m)		
≈ 5.712592027 (m)		
Construct $EM \perp CF$ such that M is the foot of perpendicular.		
then $EM = 6\sin 50°$ (m)	1M	
≈ 4.596266659 (m)		
The area of the shadow $DCFE$		
$= (\dfrac{3}{\tan 35°} + \dfrac{4}{\tan 35°}) \times \dfrac{6\sin 50°}{2}$	1M+1A	$(4.28444402 + 5.712592027) \times \dfrac{4.596266659}{2}$
≈ 22.97452173 (m)		$= 22.97452174$
≈ 23.0 (m^2)	1A	r.t. 23.0 m^2
Construct $EY // DC$ such that EY intersects CF at Y.		
then $EY = DC = 6$		
The area of the shadow $DCFE$		
= area of parallelogram $DCYE$ + area of ΔEYF		1M for the sum of two areas
$= 6 \times \dfrac{3}{\tan 35°} \times \sin 50°$	1M+1M	+1M for any one term correct
$+ \dfrac{1}{2}(\dfrac{4}{\tan 35°} - \dfrac{3}{\tan 35°}) \times 6\sin(180° - 50°)$	+1A	
≈ 22.97452173		
≈ 23.0 (m^2)	1A	r.t. 23.0 m^2
Join CE. Consider ΔDCE.		
By cosine formula, we have	1M	
$CE^2 = DC^2 + DE^2 - 2(DC)(DE)\cos \angle CDE$		
$CE^2 = 6^2 + (\dfrac{3}{\tan 35°})^2 - 2(6)(\dfrac{3}{\tan 35°})\cos 50°$		either one
≈ 21.3086102		
$CE \approx 4.616125019$ (m)		
By sine formula, we have		
$\dfrac{\sin \angle DCE}{DE} = \dfrac{\sin \angle CDE}{CE}$		
$\dfrac{\sin \angle DCE}{4.28444402} \approx \dfrac{\sin 50°}{4.616125019}$		
$\angle DCE \approx 45.31650667°$		
$\angle ECF \approx 180° - 50° - 45.31650667°$		
$= 84.68349333°$		
The area of the shadow $DCFE$		
= area of ΔDCE + area of ΔCEF		
$\approx \dfrac{1}{2} \times 6 \times 4.28444402\sin 50°$	1M+1A	
$+ \dfrac{1}{2} \times 4.616125019 \times 5.712592027\sin 84.68349333°$		
≈ 22.97452173		
≈ 23.0 (m^2)	1A	r.t. 23.0 m^2
	---------(5)	

Solution	Marks	Remarks
(b) The area of the shadow $= (DE + CF) \times \dfrac{DC \sin(90° - \theta°)}{2}$		
When $40 < \theta < 90$, $90° - \theta°$ is less than $50°$. Note that DE, CF and DC are constants, and $\sin(90° - \theta°) < \sin 50°$.	1M	
Thus, the area of the shadow is less than the area obtained in (a).	1A ---------(2)	f.t.
(c)		
		
The shortest path is EM.	1M	
In rt.$\triangle PQR$, $QR = \dfrac{1.8}{\tan 35°}$	1M	$QR \approx 2.570666412$
$MF = CF - CM$ $= CF - GE$ $= CF - (DE - DG)$ $= \dfrac{4}{\tan 35°} - (\dfrac{3}{\tan 35°} - 6\cos 50°)$ ≈ 5.284873665	1M	
$\triangle EQR \sim \triangle EMF$ $\dfrac{EQ}{EM} = \dfrac{QR}{MF}$	1M	
$\dfrac{x}{6\sin 50°} \approx \dfrac{\frac{1.8}{\tan 35°}}{5.284873665}$	1A	$\dfrac{x}{6\sin 50°} \approx \dfrac{2.570666412}{5.284873665}$
$x \approx 2.235714431$ ≈ 2.24	1A ---------(6)	r.t. 2.24

學友社 2015／16 年香港模擬文憑試

Hok Yau Club Hong Kong Mock Examination 2015/16

數學 必修部分 試卷二

Mathematics (Compulsory Part) Paper 2

題號	答案	題號	答案
1.	B	26.	D
2.	C	27.	A
3.	A	28.	D
4.	D	29.	A
5.	C	30.	B
6.	A	31.	D
7.	B	32.	A
8.	D	33.	B
9.	A	34.	A
10.	C	35.	A
11.	D	36.	D
12.	B	37.	C
13.	A	38.	C
14.	D	39.	B
15.	D	40.	B
16.	B	41.	C
17.	A	42.	A
18.	B	43.	C
19.	C	44.	D
20.	D	45.	B
21.	B		
22.	C		
23.	C		
24.	D		
25.	C		

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