2016/17-ME
MATH CP
PAPER 1

HOK YAU CLUB

HONG KONG MOCK EXAMINATION 2016/17

MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

9.00 am – 11.15 am (2¼ hours)

This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1 and 3.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answer should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

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Please stick the barcode label here.

Candidate Number									

	Marker's Use Only	**Examiner's Use Only**
	Marker No.	Examiner No.
Question No.	Marks	Marks
1-2		
3-4		
5-6		
7		
8		
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11		
12		
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15		
16		
17		
18		
19		
20		
Total		

SECTION A (1) (35 marks)

1. Simplify $\frac{(x^4y^{-3})^2}{x^{-4}y^7}$ and express your answer with positive indices. (3 marks)

2. Make s the subject of the formula $t(2s-r)=4(s-5t)$. (3 marks)

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3. Factorize

(a) $2p^2 + pq - 6q^2$,

(b) $2p^2 + pq - 6q^2 + 9q - 6p$.

(3 marks)

4. Andy buys a toy then sells the toy to Betty at a profit of 20%. Later, Betty sells the toy to Calvin at a loss of 25%. It is given that Andy gains $\$28$.

(a) Find the price of the toy for Andy to purchase it.

(b) How much does Calvin spend on buying the toy?

(4 marks)

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5. In a kindergarten, the ratio of the number of girls to that of boys is $5 : 4$. If the number of girls is increased by 72, then the number of girls will be twice the number of boys. Find the difference of the number of girls and the number of boys. (4 marks)

6. Consider the compound inequality

$$\frac{1-4x}{2} \geq 9 \quad \text{or} \quad 5-x<0 \quad \cdots\cdots(*) \quad .$$

(a) Solve (*) .

(b) Write down the greatest negative integer satisfying (*) .

(4 marks)

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7. The coordinates of the points P and Q are $(-4, 5)$ and $(4, -8)$ respectively. P is rotated anti-clockwise about the origin O through $270°$ to P'. Q is translated leftwards by k units to Q'.

(a) Write down the coordinates of P'.

(b) Suppose $P'OQ'$ is a straight line. Find k.

(4 marks)

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8. The stem-and-leaf diagram below shows the distribution of the scores (in marks) of 20 students in a Mathematics Test.

Stem (10 marks)	Leaf (1 mark)
5	*a* 3 7
6	0 2 3 4 5 8 9
7	1 3 6 7 8 8
8	2 4 6 *b*

It is given that the range and the mean of the score distribution are 34 marks and 70.2 marks respectively.

(a) Find a and b.

(b) If a student is randomly selected from the 20 students, find the probability that the score of the selected student in the Mathematics Test is divisible by 4 .

(5 marks)

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9. In Figure 1 , $ABCD$ is a parallelogram. E is a point lying on AB produced . F is a point lying on CD produced . Also, $BE = DF$.

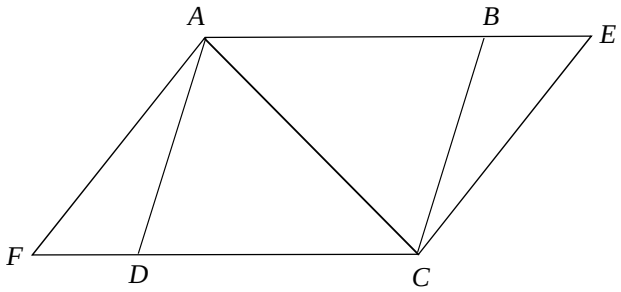

Figure 1

(a) Prove that $\Delta ACE \cong \Delta CAF$.

(b) Suppose $AF = 20\text{ cm}$, $AC = 15\text{ cm}$, $BE = 10\text{ cm}$ and $\angle ACB = \angle ABC$. Find the area of ΔACE .

(5 marks)

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SECTION A (2) (35 marks)

10. The total publishing cost of books is $\$C$. It is given that C is the sum of two parts, one part is a constant and the other part varies directly as n, where n is the number of books that are published. When $n = 4\,000$, $C = 152\,000$; when $n = 6\,000$, $C = 222\,000$.

(a) When the publishing cost per book is \$40, find the number of books that are published.

(4 marks)

(b) Now, 5 000 books are published and the selling price of each book is \$42. The publisher claims that there is a loss even when all the published books are sold. Do you agree? Explain your answer. (2 marks)

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11. Consider the circle $C: x^2 + y^2 - 12x - 16y - 69 = 0$. Let X be the centre of C .

(a) Write down the coordinates of X and the radius of C . (2 marks)

(b) The straight line $L: 3x - 4y - 11 = 0$ and C intersect at two points A and B . A moving point P is equidistant from A and B . Denote the locus of P by Γ . Given that Γ cuts the x-axis and the y-axis at H and K respectively. Denote the origin by O . Someone claims that the area of ΔOHK is smaller than $\frac{1}{4}$ of the area of circle C . Is the claim correct? Explain your answer. (4 marks)

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12. The following table shows the distribution of the numbers of group members joining the package tour provided by a travel agent. It is given that the median of the numbers of group members is 2.5 . Also, $a > 10$, $3 < c < 8$ and there are 28 groups in which the number of group members is less than or equal to 3 .

Number of group members	1	2	3	4	5
Number of groups	9	a	b	c	5

(a) Find a , b and c . (3 marks)

(b) Two more groups now join the package tour. It is found that the numbers of group members of these two groups are different and the range of the numbers of group members remains unchanged. Find the least possible value and the greatest possible value of the standard deviation of the numbers of group members. (4 marks)

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13. Figure 2 shows a vessel which is made by putting a cylinder on the top of a frustum . The height of the vessel is 31 cm , the upper base radius and the lower base radius of the frustum are 10 cm and 15 cm respectively. It is given that the capacities of the cylinder and the frustum are the same.

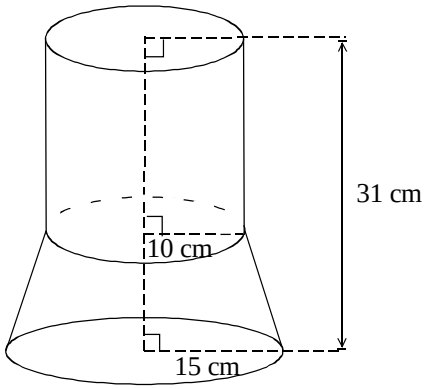

Figure 2

(a) Find the capacity of the frustum. (4 marks)

(b) 0.007 m^3 of water is now poured into the vessel. David claims that the depth of the water is greater than half of the height of the vessel. Is the claim correct? Explain your answer. (3 marks)

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14. Let $p(x)$ be a polynomial with the degree of 3 . It is given that $p(-2) = p(3) = 0$, $p(1) = -18$ and $p(2) = -20$.

(a) Find $p(x)$. (5 marks)

(b) How many rational roots does the equation $p(x) = 3x - 9$ have? Explain your answer.

(4 marks)

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Page total

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SECTION B (35 marks)

15. The mean and the standard deviation of the test scores obtained by a class of students in a test are 38 marks and 10 marks respectively. Due to the poor performance, the test score of each student is adjusted such that each score is increased by 10 % and then extra 8 marks are added. The original standard score of Kelly in the test is -0.1. She claims that her standard score will be positive after the score adjustment. Do you agree? Explain your answer. (3 marks)

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16. There are three departments A , B and C in a company. It is given that there are 4 supervisors, 4 supervisors and 5 supervisors in each of these three departments respectively. 7 people are randomly selected from the 13 supervisors to form a committee.

(a) Find the probability that the numbers of supervisors from departments A and B are the same in the committee. (2 marks)

(b) It is given that the numbers of supervisors from departments A and B are the same in the committee, find the probability that the number of supervisors from department C in the committee is the greatest. (2 marks)

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17. The 1st term and the 6th term of a geometric sequence are 8 and 1 944 respectively. Find

(a) the common ratio of the sequence, (2 marks)

(b) the least value of n such that the sum of the first n terms of the sequence is greater than 100 000 000 . (3 marks)

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18. Let $\mathrm{f}(x) = -\frac{1}{2}x^2 + \frac{1}{4}x + 1$.

(a) Using the method of completing the square, find the coordinates of the vertex of the graph of $y = \mathrm{f}(x)$. (2 marks)

(b) It is given that the straight line $y = c$ $(c > 0)$ and the graph of $y = \mathrm{f}(x)$ intersect at two points P and Q, and the length of the line segment PQ is $\frac{1}{2}c$. Find c. (3 marks)

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19. Figure 3(a) shows the base $ABCD$ of a pyramid. It is given that $AB = BC$, $AD = DC = 2\sqrt{6}$ cm, $\angle ABC = 90°$ and $\angle BAD = 75°$.

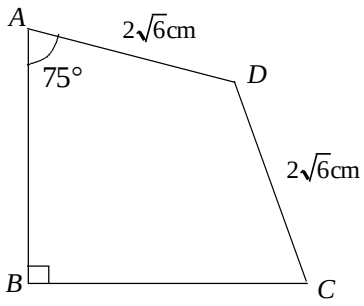

Figure 3(a)

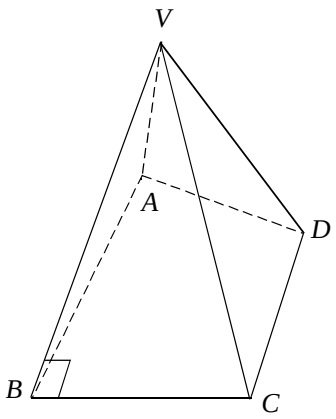

Figure 3(b)

(a) Find AB. (2 marks)

(b) Figure 3(b) shows a pyramid $VABCD$ with base $ABCD$. It is given that VAB is an equilateral triangle and $\angle VBC = 90°$.

(i) Find VD.

(ii) Let N be a point lying on DC such that BN is perpendicular to DC. Cindy claims that the angle between the plane VCD and the plane $ABCD$ is $\angle VNB$. Is the claim correct? Explain your answer.

(5 marks)

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20. In Figure 4 , ABC is an acute-angled triangle. Denote the centroid and the orthocenter of ΔABC by G and H respectively. BH is produced to meet AC at D , CH is produced to meet AB at E , AG is produced to meet BC at M . Suppose N is the mid-point of ED .

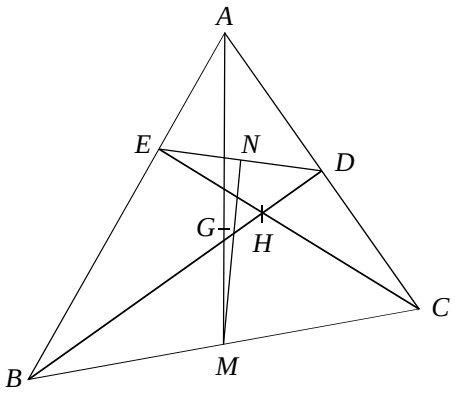

Figure 4

(a) (i) Prove that B , C , D and E are concyclic. Also, prove that the centre of the circle passing through these four points is M .

(ii) Someone claims that $MN \perp ED$. Is the claim correct? Explain your answer.

(4 marks)

(b) A rectangular coordinate system is introduced so that the coordinates of D and E are $(6, 3)$ and $(4, 4)$ respectively and the equation of BC is $x - 7y = 0$.

(i) Find the coordinates of point C .

(ii) It is given that the tangent to the circle $BCDE$ at point C cuts the x-axis and the y-axis at two points P and Q respectively, find the radius of the inscribed circle of ΔOPQ , where O is the origin.

(7 marks)

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