2017/18-ME
MATH CP
PAPER 1

HOK YAU CLUB

HONG KONG MOCK EXAMINATION 2017/18

MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

9.00 am — 11.15 am (2¼ hours)

This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1 and 3.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

Please stick the barcode label here.

Candidate Number									

	Marker's Use Only	Examiner's Use Only
	Marker No.	Examiner No.
Question No.	Marks	Marks
1-2		
3-4		
5-6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
Total		

SECTION A (1) (35 marks)

1. Simplify $\dfrac{(a^{-2})^3}{(a^4b^{-1})^2}$ and express your answer with positive indices . (3 marks)

2. Make q the subject of the formula $r = 3 + \dfrac{5p}{q-2}$. (3 marks)

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3. Factorize

(a) $9x^2 - 12xy + 4y^2$,

(b) $9x^2 - 12xy + 4y^2 - 21x + 14y$.

(3 marks)

4. A group of teachers and students visited the Science Museum , the sum of money for the admission tickets sold is \$2208 . It is given that the prices of a teacher ticket and a student ticket are \$72 and \$60 respectively , and the ratio of the number of teachers to that of students is $1:8$. Find the total number of teachers and students visited the Science Museum . (4 marks)

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5. Consider the compound inequality

$$\frac{2(x+3)}{5} > -2x-9 \quad \text{or} \quad 2-x \le 5 \quad \cdots\cdots(*) \ .$$

(a) Solve (*) .

(b) Write down the least negative integer satisfying (*) .

(4 marks)

6. The coordinates of the points A and B are $(-4,3)$ and $(4,-4)$ respectively . A is rotated clockwise about the origin through $270°$ to A' . B is translated 2 units to the right and 12 units upwards to B' .

(a) Write down the coordinates of A' and B' .

(b) Prove that $A'OB'$ is a straight line , where O is the origin .

(4 marks)

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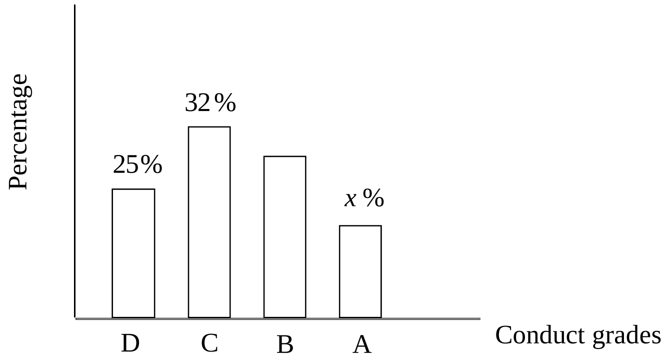
7. The bar chart below shows the distribution of the conduct grades of the students in a school .



If a student is randomly selected from the school , the probability that the selected student gets conduct in grade A or D is $\frac{2}{5}$.

(a) Find x .

(b) There are 350 students got conduct in grade B in the school . Find the number of students in the school .

(4 marks)

Answers written in the margins will not be marked.

8. It is given that $f(x)$ is the sum of two parts, one part is a constant and the other part varies as x^2. Suppose that $f(2) = 23$ and $f(7) = 563$.

(a) Find $f(x)$.

(b) Solve the equation $f(x) = 20x$.

(5 marks)

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9. A pill is termed *standard* if its weight is measured as 500 mg correct to the nearest 10 mg .

(a) Find the least possible weight of a *standard* pill .

(b) A pharmacist claims that the total weight of 160 *standard* pills can be measured as 79.4 g correct to the nearest 0.1 g . Do you agree ? Explain your answer .

(5 marks)

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SECTION A (2) (35 marks)

10. The stem-and-leaf diagram below shows the distribution of the weights (in kg) of a group of students .

Stem (tens)	Leaf (units)
4	a 2 5 5 6 8 8
5	0 3 4 5 6 8
6	2 8 b

It is given that the mean and the range of the above distribution are 52.5 kg and 28 kg respectively .

(a) Find the standard deviation of the above distribution . (4 marks)

(b) If two students are randomly selected from the group , find the probability that the difference of the weights of the selected students exceeds 20 kg . (2 marks)

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11. In Figure 1, $ABCD$ is a rhombus. M and N are points lying on BC and CD respectively, such that $AM \perp BC$ and $AN \perp CD$. AM and AN cut BD at E and F respectively.

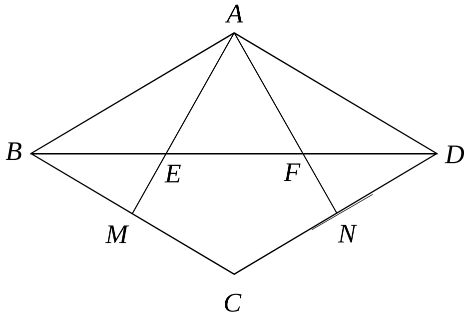

Figure 1

(a) Prove that $\Delta ABM \cong \Delta ADN$. (2 marks)

(b) Explain why $AE = AF$. (2 marks)

(c) If $AE = 12\text{ cm}$ and $\angle BAE = 20°$, find EF. (3 marks)

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12. The paper sector as shown in Figure 2(a) is folded to form an inverted right circular conical vessel as shown in Figure 2(b) . Some water is poured into the vessel and the depth of the water is $\frac{2}{3}$ the height of the vessel .

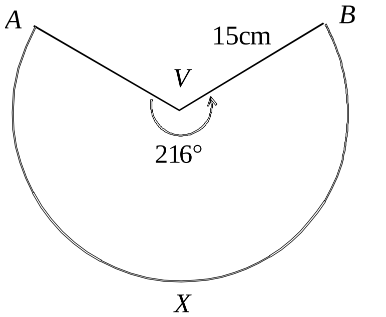

Figure 2(a)

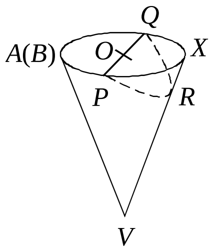

Figure 2(b)

(a) Find the area of the wet part of the vessel in terms of π . (4 marks)

(b) Let PQ be the base diameter of the vessel with O as the centre . An ant crawls around the curved surface of the vessel from P to Q . Is it possible that the distance crawled by the ant is less than the length of the semi-circle PXQ ? Explain your answer . (3 marks)

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13. Let $f(x)=6x^3+5x^2+kx-8$. When $f(x)$ is divided by $3x^2+ax+2$, the quotient and the remainder are $2x+b$ and $cx-6$ respectively , where a , b and c are constants .

(a) Find a . (3 marks)

(b) Let $g(x)$ be a polynomial with the degree of three such that when $g(x)$ is divided by $3x^2+ax+2$, the remainder is $cx-6$.

(i) Prove that $f(x)-g(x)$ is divisible by $3x^2+ax+2$.

(ii) Someone claims that all the roots of the equation $f(x)-g(x)=0$ are real numbers . Do you agree ? Explain your answer .

(4 marks)

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14. The equation of the circle C is $x^2+y^2+2x-8y-83=0$. Denote the centre of C by A .

(a) Find the coordinates of A and the radius of C . (2 marks)

(b) The coordinates of the point B are $(23,22)$. P is a moving point in the rectangular coordinate plane such that $PA=PB$. Denote the locus of P by Γ.

(i) Find the equation of Γ.

(ii) Let H be a point on C such that it is nearest to Γ, K be a point on Γ such that it is nearest to C , and R be a point lying on the positive x-axis . Find the ratio of the areas of ΔRAH to ΔRHK .

(6 marks)

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SECTION B (35 marks)

15. Let a and b be constants . Denote the graph of $y=\dfrac{a}{b^x}$ by G . It is given that G passes through the points $(1,3)$ and $(4,\dfrac{1}{9})$. Express x in terms of y . (4 marks)

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16. There are 8 balls of different colours in a bag . 4 balls are randomly drawn from the bag one by one with replacement and their colours are recorded .

(a) Find the probability that the two balls drawn are of the same colour and another two balls drawn are of another same colour . (2 marks)

(b) Find the probability that the four balls drawn are of two colours only . (3 marks)

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17. For any positive integer n , let $\mathrm{T}(n) = 3^{4n-1}$.

(a) Express $\mathrm{T}(1) + \mathrm{T}(2) + \mathrm{T}(3) + \cdots + \mathrm{T}(n)$ in terms of n . (2 marks)

(b) Find the least value of n such that $\mathrm{T}(1)\mathrm{T}(2)\mathrm{T}(3)\cdots\mathrm{T}(n) \geq 3^{630}$. (3 marks)

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18. The equation of the parabola Γ is $y = 2x^2 - 4kx + k^2 - 1$, where k is a real number . P is the vertex of the graph of Γ. Denote the straight line $y = 15$ by L .

(a) Using the method of completing the square , express the coordinates of P in terms of k .

(2 marks)

(b) Prove that L and Γ intersect at two distinct points . (2 marks)

(c) The points of intersection of L and Γ are A and B .

(i) Let a and b be the x-coordinates of A and B respectively . Express $(a-b)^2$ in terms of k .

(ii) Is it possible that the area of ΔPAB is less than 40 ? Explain your answer .

(4 marks)

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19. Figure 3(a) shows a triangular wooden board ABC . $\angle BAC = 60°$ and $AC = 2\text{ m}$. The wooden board is inclined with side BC on the horizontal ground . The inclinations of sides AB and AC to the horizontal ground are $45°$ and $30°$ respectively . $A'BC$ is the shadow of the wooden board cast on the horizontal ground and AA' is a vertical line .

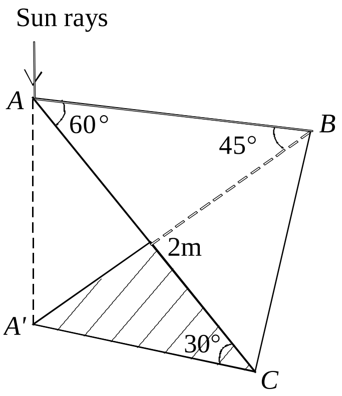

Figure 3(a)

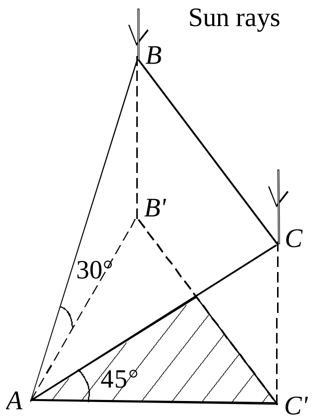

Figure 3(b)

(a) Find the lengths of AB and BC . (3 marks)

(b) The wooden board described in Figure 3(a) is inclined in another way with vertex A on the horizontal ground as shown in Figure 3(b) . The inclinations of sides AB and AC to the horizontal ground are $30°$ and $45°$ respectively . $AB'C'$ is the shadow of the wooden board cast on the horizontal ground . BB' and CC' are vertical lines .

(i) Find the length of $B'C'$.

(ii) Find $\angle B'AC'$. Hence , or otherwise , find the area of the shadow $B'AC'$.

(iii) A student claims that the angle of inclination of the wooden board ABC to the horizontal ground in Figure 3(b) is less than $45°$. Do you agree ? Explain your answer .

(10 marks)

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